

Exercise 48

Find the absolute maximum and absolute minimum values of f on the given interval.

$$f(x) = 5 + 54x - 2x^3, \quad [0, 4]$$

Solution

Take the derivative of the function.

$$\begin{aligned} f'(x) &= \frac{d}{dx}(5 + 54x - 2x^3) \\ &= 5(0) + 54(1) - 2(3x^2) \\ &= 54 - 6x^2 \end{aligned}$$

Set $f'(x) = 0$ and solve for x .

$$54 - 6x^2 = 0$$

$$6x^2 = 54$$

$$x^2 = 9$$

$$x = \pm 3$$

$x = 3$ is within $[0, 4]$, so evaluate f here.

$$f(3) = 5 + 54(3) - 2(3)^3 = 113 \quad (\text{absolute maximum})$$

Now evaluate the function at the endpoints of the interval.

$$f(0) = 5 + 54(0) - 2(0)^3 = 5 \quad (\text{absolute minimum})$$

$$f(4) = 5 + 54(4) - 2(4)^3 = 93$$

The smallest and largest of these numbers are the absolute minimum and maximum, respectively, over the interval $[0, 4]$.

The graph of the function below illustrates these results.

